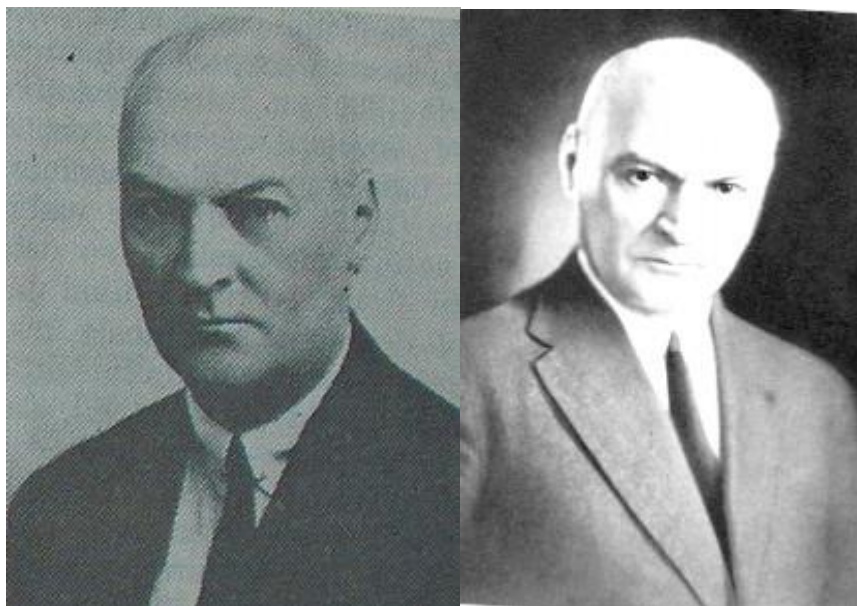# DIMITRIE POMPEIU

We give a short presentation of the work of Dimitrie Pompeiu, following [20], [21] and the other references. He was a full member of the Romanian Academy since 1934 and of the Academy of Romanian Scientists since 1943.

On March 31-st, 1905, Dimitrie Pompeiu (1873-1954), a distinguished Romanian mathematician, defended at the Faculte de Sciences de Paris his Ph.D. work *Sur la continuite des fonctions de variables complexes* . The comission was chaired by H. Poincare and included G. Koenigs and E. Goursat and the work was published the same year in [14]. P. Montel, a wellknown French mathematician, has appreciated the remarkable contribution of Pompeiu's thesis by the words: *Pour un coup d'essai, c'est un coup de maître,* [1]. Ideas, notions and results discussed in the thesis of Pompeiu have attracted the interest of important contemporary mathematicians: P. Painleve, A. Denjoy, F. Hausdorff and is now a part of the universal mathematical heritage. Pompeiu studied a problem formulated by Painleve [13] already in 1897, concerning the singularities of uniform analytic functions. At that time, it was generally admitted that a uniform analytic function cannot be continuously extended on the set of its singularities. However Pompeiu could construct examples of such functions which are continuous on the set of their singularities and this set has positive measure. The controversy that followed was solved in 1909 by Denjoy who confirmed that the arguments of Pompeiu are correct. This was a turning point in the theory of uniform analytic functions and Pompeiu was considered as the best specialist in this field, at the beginning of the last century [1]. Pompeiu used an example due to A. Koepcke (pag.45 in [14]) which was rather complicated. Some years later, in [16], he constructed much simpler examples of the same type, that is real functions with bounded derivatives that have roots in any interval and are not identically zero in some interval. Such functions are now called Pompeiu functions and are included in many monographs on real functions.

Very well documented presentations of this moment from the history of mathematics may be found in [8], [9]. In his arguments in the complex plane, Pompeiu needed a notion of distance between closed curves. It is to be remarked that quite in the same period, M. Frechet introduced in his Ph.D. thesis (published a little bit later in [3]) the distance between two elements and the axioms of metric spaces. This is the context in which D. Pompeiu defines in 1905 the notions of ecart and ecart mutuel between two sets ([14], p.17 and 18). Consequently, Pompeiu is also considered as one of the founders of the theory of hyperspaces, see McAllister [10]. The thesis of D. Pompeiu is the birth certicate of the notion of distance between sets and its importance was immediately remarked by the contemporary mathematicians. Already in 1905, in his report for the Jahrbuch fuer Mathematik, on p.455 A. Gutzmer calls the Pompeiu distance *Einfuehrung eines neuen Begriffes der Mengenlehre* (the introduction of a new concept in set theory). In 1914,in his famous book [5], Hausdorff studies the notion of set distance, in

the natural setting of metric spaces and with a small modication (the sum si replaced by the maximum). He devotes a consistent paragraph to the study of the properties of the distance and he quotes correctly Pompeiu [5] on p. 463, as the author of this notion. Moreover he also establishes that the distances defined with the „sum" or with the „max" give the same toplogy, that is they are equivalent. The same considerations may be found in his book from 1927, [6], where

Pompeiu is quoted at page 280. It is to be noted that at that time, the main text of a book did not include any reference to the origin of a subject and all these references were concentrated in a short appendix at the end (Quellenangabe). Probably, due to this reason, many mathematicians didn't notice the reference to Pompeiu's work [14] and the set distance is frequently termed as the Hausdorff distance. Other names used in the scientific literature are Pompeiu distance or Pompeiu-Hausdorff distance, Hausdorff-Pompeiu distance. The great Polish mathematician C. Kuratowski, in his treatise [7], on p. 106 mentions both the thesis of Pompeiu from 1905 and the book of Hausdorff from 1914. This is done in a footnote, on the page where the notion is introduced and is completely accessible to the reader.

In 1978, McAllister [10] published a remarkable historical study devoted to the first 50 years in the theory of hyperspaces. The role of Pompeiu's thesis is underlined clearly: *„who [Pompeiu] may with some justice be said to have invented hyperspaces, and Hausdorff's use of them in 1914 in his treatise Grundzuege der Mengenlehre had made them very well known" (p. 310) and similarly on p. 311 „I have found no evidence of the Hausdorff metric itself before Pompeiu's thesis"*. There are many notions of set distance which are not equivalent with the Hausdorff-Pompeiu distance. For instance, one can define the distance between two Lebesgue measurable subsets in an Euclidean space as the measure of their symmetric difference. However, the Hausdorff-Pompeiu metric seems to be the only one with a remarkable compactness property: if (A_n) is a sequence of compacts, bounded with respect to n, then there is a compact subset A and a subsequence again indexed by n, such that $d(A_n;A)$ converges to 0 in this metric. This property makes the Hausdorff-Pompeiu distance a fundamental notion in the study of the topologies on families of subsets and in the modern theory of shape optimization (optimal design). The interested reader may consult the recent monograph [11] and its references for a survey of the mathematical literature on geometric optimization.

2 The Pompeiu conjecture

The whole mathematical work of Pompeiu is characterized by a profound originality, by the introduction of many fruitful ideas and methods. We briefly recall the definition in 1912 of the areolar derivative (see [15]), further developed by M. Nicolescu, Gh. Calugareanu, N. Cioranescu, N. Teodorescu, Gr. Moisil and others. This is in fact a fundamental◌ operator from complex analysis. In this context, Pompeiu also proved the Cauchy -Pompeiu formula which appeared here for the first time, [9]. Probably the best known paper of Pompeiu is his Note [17] from 1929 (recent estimates indicate almost one thousand articles quoting it). Here one can find the famous Pompeiu conjecture, still unsolved completely : Let f be a continuous function in the plane and D a compact subset such the integral on any compact set obtained from D by rigid motions, is null.

Then, is it true that f is null in the plane ? For the case when D is a disk, there are counterexamples [4] of the form $f(x; y) = sin(ax+by)$ with $a; b$ appropriately chosen real numbers. For any other domains in the plane, the answer seems to be positive although just some special cases are solved [19]. We underline the fundamental character in the mathematical analysis of this property (if proved) and its relationship with the Schiffer conjecture (formulated later) concerning the eigenvalues of the Laplace operator with Cauchy conditions on the boundary, [2], [18].

O. Onicescu, a student of Pompeiu and a wellknown Romanian mathematician, said that the creations of Dimitrie Pompeiu are *„simple, plastic,global and full of significance in the world of science"*, [12].

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