UNIVERSITATEA NATIONALA DE STIINTA SI TEHNOLOGIE POLITEHNICA BUCURESTI

FACULTATEA DE ȘTIINȚE APLICATE

Buna-definire, rezultate de existență și caracterizare a soluțiilor pentru anumite probleme variaționale

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Cuprins

1. Abstract

- Tema de cercetare, în care a fost înaintată propunerea de proiect, este unul dintre punctele importante şi de actualitate ale comunității relevante de cercetare. Aceasta include metode de optimizare robustă şi incertă, și teoria controlului optimal cu aplicații în inginerie, sisteme fizice și socio-economice.

2. Rezultate obţinute

În cadrul acestui proiect, în perioada 30.06.2023-27.11.2023, au fost obţinute următoarele rezultate:

A. Articole publicate (selecţie)

- Y. Guo, G. Ye, W. Liu, D. Zhao, **S. Treanţă**, *Solving nonsmooth interval optimization problems based on interval-valued symmetric invexity*, Chaos, Solitons and Fractals, Vol. 174, 113834, 2023; DOI: 10.1016/j.chaos.2023.113834; WOS: 001053527500001; ISSN: 0960-0779.
- **S. Treanţă**, T. Saeed, *Characterization Results of Solution Sets Associated with Multiple-Objective Fractional Optimal Control Problems,* Mathematics, Vol. 11, No. 14, 3191, 2023; DOI: 10.3390/math11143191; WOS: 001071425400001; eISSN: 2227-7390.
- M. Ciontescu, **S. Treanţă**, *On some connections between interval-valued variational control problems and the associated inequalities,* Results in Control and Optimization, Vol. 12, 100300, 2023; DOI: 10.1016/j.rico.2023.100300; eISSN: 2666-7207.

- *Chaos, Solitons and Fractals* (Q1 – zona roşie) are *factorul de impact 7.8* (ocupă PRIMA POZIŢIE în domeniul MATHEMATICAL PHYSICS)
- *Mathematics* (Q1 – zona roşie) are *factorul de impact 2.592*
- *Results in Control and Optimization* (Elsevier)

B. Articole acceptate

- **S. Treanţă**, J.C. Yao, *Robust variational inequalities governed by curvilinear integral functionals,* Journal of Nonlinear and Variational Analysis, 2023.

- *Journal of Nonlinear and Variational Analysis* (Q1 – zona roşie) are *factorul de impact 2.175*.

C. Articole/capitole de carte trimise spre publicare

- **S. Treanţă**, N. Abdulaleem, *On Variational Derivative and Controlled Variational Inequalities*, Editor: S.K. Mishra; Springer, 2023.
- C. Cebuc, **S. Treanţă**, *On some optimization problems governed by approximately star-shaped functionals,* Journal of Optimization Theory and Applications, 2023.
- F. Shi, G. Ye, W. Liu, D. Zhao, **S. Treanţă**, *Lagrangian dual theory and stability analysis for fuzzy optimization problems,* Information Sciences, 2023.
- C. Marghescu, **S. Treanţă**, *Control problems driven by approximately pseudo-convex multiple integral functionals,* Mathematical Control and Related Fields, 2023.
- C. Cebuc, **S. Treanţă**, *On some multiple-objective optimization problems and generalized vector control inequalities,* Journal of Mathematical Analysis and Applications, 2023.
- **S. Treanţă**, V. Singh, S.K. Mishra, *Some characterization results on a class of variational control inequalities,* Journal of Computational and Applied Mathematics, 2023.

- *Springer* este o editură internaţională de înaltă calitate ştiinţifică.
- *Journal of Optimization Theory and Applications* (Q2 – zona galbenă) are *factorul de impact 1.9*
- *Information Sciences* (Q1 – zona roşie) are *factorul de impact 8.1*
- *Mathematical Control and Related Fields* (Q2 – zona galbenă) are *factorul de impact 1.2*
- *Journal of Mathematical Analysis and Applications* (Q1 – zona roşie) are *factorul de impact 1.583*
- *Journal of Computational and Applied Mathematics* (Q1 – zona roşie) are *factorul de impact 2.4*

3. Detalierea rezultatelor

- Y. Guo, G. Ye, W. Liu, D. Zhao, **S. Treanţă**, *Solving nonsmooth interval optimization problems based on interval-valued symmetric invexity*, Chaos, Solitons and Fractals, Vol. 174, 113834, 2023; DOI: 10.1016/j.chaos.2023.113834; WOS: 001053527500001; ISSN: 0960-0779.

Această lucrare se concentrează pe o problemă de optimizare neconvexă pe interval. Pentru aceasta, propunem invexitatea simetrică cu valori pe interval, pseudo-invexitatea simetrică cu valorile pe interval și cvasi-invexitatea simetrică cu valorile pe interval în ceea ce privește funcțiile simetrice cu valori pe interval gH-diferențiabile. Sunt discutate și câteva proprietăți importante ale acestor convexități generalizate. Prin utilizarea acestor noi concepte, stabilim condiții suficiente Karush–Kuhn–Tucker pentru problema luată în considerare. În plus, problemele duale de tip Wolfe și Mond-Weir sunt asociate și au fost obținute rezultate de dualitate slabă, puternică și strict inversă. În cele din urmă, aplicăm teoria dezvoltată la o problemă de clasificare binară a datelor pe interval prin mașină vector suport.

$$\mathbb{I} = \{a = [\underline{a}, \overline{a}] | \underline{a}, \overline{a} \in \mathbb{R} \text{ and } \underline{a} \leq \overline{a}\}.$$

A partial order relation "$\preceq$" of two intervals a and b in $\mathbb{I}$ is determined by

$$[\underline{a}, \overline{a}] \preceq [\underline{b}, \overline{b}] \Leftrightarrow \underline{a} \leq \underline{b},\ \overline{a} \leq \overline{b};$$

$$[\underline{a}, \overline{a}] \prec [\underline{b}, \overline{b}] \Leftrightarrow [\underline{a}, \overline{a}] \preceq [\underline{b}, \overline{b}], \text{and } [\underline{a}, \overline{a}] \neq [\underline{b}, \overline{b}].$$

In [35], the concept of gH-difference has been given as below:

$$a \ominus_{gH} b = \left[\min\{\underline{a} - \underline{b}, \overline{a} - \overline{b}\},\ \max\{\underline{a} - \underline{b}, \overline{a} - \overline{b}\}\right]. \tag{2.1}$$

Definition 2.2 ([31]). Let $F : \mathcal{K} \to \mathbb{I}$ be an IVF, where $\mathcal{K} \subseteq \mathbb{R}$ is an open set. It is said that F is sgH-differentiable at $\breve{t} \in \mathcal{K}$, if there is $F^{s}(\breve{t}) \in \mathbb{I}$ so that

$$\lim_{\varrho \to 0} \frac{1}{2\varrho}\Big(F(\breve{t}+\varrho) \ominus_{gH} F(\breve{t}-\varrho)\Big) = F^{s}(\breve{t}). \tag{2.2}$$

Definition 2.6 ([14]). Let $F : D \to \mathbb{I}$ be an IVF, where $D \subseteq \mathbb{R}^{n}$ is a convex open set. It is said that F is LU-convex at $\breve{t} \in D$ if

$$F\big(\imath\breve{t} + (1-\imath)\breve{\gamma}\big) \preceq \imath F(\breve{t}) + (1-\imath)F(\breve{\gamma}),$$

for any $\imath \in [0,1]$ and $\breve{\gamma} \in D$.

Theorem 2.8 (*Characterization of LU-convexity*). *Let the IVF $F : \mathcal{D} \to \mathbb{I}$ be sgH-differentiable, where $\mathcal{D} \subseteq \mathbb{R}^n$ is a convex open set. If F is LU-convex on $\mathcal{D}$, then we get*

$$(\check{t} - \check{\gamma})^T \nabla^s F(\check{\gamma}) \preceq F(\check{t}) \ominus_{gH} F(\check{\gamma}), \ \forall \check{t}, \check{\gamma} \in \mathcal{D}. \tag{2.4}$$

Definition 3.1 (*Interval-valued Symmetric Invexity*). Let $F : \mathcal{D} \to \mathbb{I}$ be an IVF. It is said that F is (strictly) interval-valued symmetric invex at $t^* \in \mathcal{D}$ w.r.t. η, if F has sgH-differentiability at t^* and

$$F(\check{t}) \ominus_{gH} F(t^*)(\succ) \succeq \eta(\check{t}, t^*)^T \nabla^s F(t^*), \tag{3.1}$$

for each $\check{t} \in \mathcal{D}$.

Definition 3.6 (*Interval-valued Symmetric Pseudo-invexity*). Let $F : D \to \mathbb{I}$ be an IVF. It is said that F is interval-valued symmetric pseudo-invex at $t^* \in D$ w.r.t. η, if F has sgH-differentiability at t^* and

$$\eta(\check{t}, t^*)^T \nabla^s F(t^*) \succeq 0 \Rightarrow F(\check{t}) \ominus_{gH} F(t^*) \succeq 0 \tag{3.3}$$

Definition 3.10 (*Interval-valued Symmetric Quasi-invexity*). Let $F : D \to \mathbb{I}$ be an IVF. It is said that F is interval-valued symmetric quasi-invex at $t^* \in D$ w.r.t. η, if F is sgH-differentiable at t^* and

$$F(\check{t}) \ominus_{gH} F(t^*) \preceq 0 \Rightarrow \eta(\check{t}, t^*)^T \nabla^s F(t^*) \preceq 0 \tag{3.6}$$

for each $\check{t} \in D$.

$$\begin{aligned} \min \quad & F(\check{t}) & \text{(P)} \\ \text{subject to} \quad & G_i(\check{t}) \preceq 0, \quad i = 1, \ldots, l, \\ & \check{t} \in \mathcal{D}, \end{aligned}$$

where $F,\ G_i\ (i = 1, \ldots, l) : \mathcal{D} \subset \mathbb{R}^n \to \mathbb{I}$, $\mathcal{D}$ is an open invex set. We represent the collection of feasible points in (P) as

$$\Gamma_P = \{\check{t} \in \mathbb{R}^n : \check{t} \in \mathcal{D} \text{ and } G_i(\check{t}) \preceq 0,\ i = 1, \ldots, l\}$$

Theorem 4.2 (*Sufficient KKT Condition*). *Let $t^* \in \Gamma_P$. Suppose that F and $G_i,\ i = 1, \ldots, l$ are $\mathcal{IS}$-invex w.r.t. same $\eta : \mathcal{D} \times \mathcal{D} \to \mathbb{R}^n$ at t^*. If there is a scalar $\mu = (\mu_1, \ldots, \mu_l)$, so that*

$$\begin{aligned} &\nabla^s(\underline{F} + \overline{F})(t^*) + \Sigma_{i=1}^{l} \mu_i \nabla^s(\underline{G_i} + \overline{G_i})(t^*) = \mathbf{0}, \\ &\mu_i \underline{G_i}(t^*) = \mu_i \overline{G_i}(t^*) = 0,\ i = 1, \ldots, l, \end{aligned} \tag{4.1}$$

where $0 \leq \mu_i \in \mathbb{R}\ (i = 1, \ldots, l)$, then t^* is a ND solution to (P).

The Wolfe-type dual problem of (P) is

$$
\begin{aligned}
\max \quad & F(z) + \Sigma_{i=1}^{l} \mu_i G_i(z) && \text{(WDP)} \\
\text{subject to} \quad & \nabla^s(\underline{F} + \overline{F})(z) + \Sigma_{i=1}^{l} \mu_i \nabla^s(\underline{G_i} + \overline{G_i})(z) = \mathbf{0}, \\
& \mu = (\mu_1, \ldots, \mu_l) \geq \mathbf{0}, \\
& z \in \mathcal{D}.
\end{aligned}
$$

Theorem 5.2 (*Weak Duality*). *Suppose* $\check{t} \in \Gamma_P$, $(z, \mu) \in \Gamma_{WDP}$. *If* F *and* G_i $(i = 1, \ldots, l)$ *are* $\mathcal{I}S$*-invex w.r.t. same* η *at* z, *then*

$$F(\check{t}) \nprec \zeta(z, \mu). \tag{5.2}$$

Theorem 5.4 (*Strong Duality*). *Let t^* be a ND solution to* (P). *Assume that F and G_i $(i = 1, \ldots, l)$ are $\mathcal{IS}$-invex w.r.t. same η on D. If there is a scalar $\mu^* \geq 0$ such that*

$$\begin{aligned} &\nabla^s(\underline{F} + \overline{F})(t^*) + \Sigma_{i=1}^{l} \mu_i^* \nabla^s(\underline{G_i} + \overline{G_i})(t^*) = \mathbf{0}, \\ &\mu_i^* \underline{G_i}(t^*) = \mu_i^* \overline{G_i}(t^*) = 0, \ \ i = 1, \ldots, l, \end{aligned} \tag{5.7}$$

where $\mu^* = (\mu_1^*, \ldots, \mu_l^*)^T$, then (t^*, μ^*) is a ND solution to (WDP). *Further,* (P) *and* (WDP) *have the same extreme values.*

Theorem 5.5 (*Strict Converse Duality*). *Let $(z, \mu) \in \Gamma_{WDP}$ and $\check{t} \in \Gamma_P$. Assume that F is strictly $\mathcal{IS}$-invex and G_i $(i = 1, \ldots, l)$ are $\mathcal{IS}$-invex w.r.t. same η at z. If*

$$F(\check{t}) + \Sigma_{i=1}^{l} \mu_i G_i(\check{t}) \preceq \zeta(z, \mu), \tag{5.8}$$

then $\check{t} = z$.

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Această lucrare investighează unele rezultate de dualitate mixtă pentru o clasă de probleme de control optimal fracțional cu obiectiv multiplu. Mai precis, luând în considerare dualitățile de tip Wolfe și Mond-Weir, formulăm o problemă duală robustă de tip mixt și, sub ipoteze de convexitate adecvate ale funcționalelor implicate, stabilim unele rezultate de echivalență între mulţimile de soluții ale modelelor luate în considerare. În esență, investigăm rezultate de dualitate robustă slabă, puternică și strict inversă. Rezultatele de dualitate robustă pentru astfel de probleme sunt noi în literatura de specialitate.

$$(\mathcal{P}) \qquad \min_{(\lambda,\pi)} \left\{ \frac{\int_S h(\Upsilon,\varsigma)dt}{\int_S z(\Upsilon,\gamma)dt} := \left(\frac{\int_S h_1(\Upsilon,\varsigma_1)dt}{\int_S z_1(\Upsilon,\gamma_1)dt}, \ldots, \frac{\int_S h_p(\Upsilon,\varsigma_p)dt}{\int_S z_p(\Upsilon,\gamma_p)dt} \right) \right\}$$

subject to

$$f(\Upsilon, \lambda_\alpha(t), \sigma) \leqq 0,$$

$$g(\Upsilon, \lambda_\alpha(t), \delta) := \lambda_\alpha(t) - \Theta_\alpha(\Upsilon, \delta) = 0, \quad \alpha = \overline{1,p}$$

$$t \in S,\ \lambda(t_0) = \lambda_0,\ \lambda(t_1) = \lambda_1,$$

where

$$h_\epsilon : S \times A \times B \times \mathcal{G}_\epsilon \to \mathbb{R},\ \epsilon = \overline{1,p}, \quad h = (h_1, \ldots, h_p),$$

$$z_\epsilon : S \times A \times B \times \mathcal{Q}_\epsilon \to \mathbb{R},\ \epsilon = \overline{1,p}, \quad z = (z_1, \ldots, z_p),$$

$$f_l : J^1\left(S, \mathbb{R}^q\right) \times B \times \mathcal{T}_l \to \mathbb{R},\ l = \overline{1,m}, \quad f = (f_1, \ldots, f_m),$$

$$g_s : J^1\left(S, \mathbb{R}^q\right) \times B \times \mathcal{M}_s \to \mathbb{R}, s = \overline{1,n}, \quad g = (g_1, \ldots, g_n),$$

are C^1-class functionals (almost everywhere); the jet bundle of first-order associated with S and $\mathbb{R}^q$ is stated as $J^1\left(S, \mathbb{R}^q\right)$; also, we assume $\int_S z_\epsilon(\Upsilon, \gamma_\epsilon)dt > 0, \epsilon = \overline{1,p}$, and $\varsigma = (\varsigma_\epsilon), \gamma = (\gamma_\epsilon), \sigma = (\sigma_l)$, and $\delta = (\delta_s)$ represent the uncertainty parameters of the compact convex sets $\mathcal{G} = (\mathcal{G}_\epsilon) \subset \mathbb{R}^p$, $\mathcal{Q} = (\mathcal{Q}_\epsilon) \subset \mathbb{R}^p$, $\mathcal{T} = (\mathcal{T}_l) \subset \mathbb{R}^m$, and $\mathcal{M} = (\mathcal{M}_s) \subset \mathbb{R}^n$.

Next, we associate a mixed robust dual model for $(\mathcal{P})$, as follows:

$$(m\mathcal{D}-\mathcal{P}) \qquad \max_{(\iota(\cdot),\kappa(\cdot))} \int_S \{ \left[h(\Pi,\varsigma) - Q^0 z(\Pi,\gamma) \right] + \rho^T f(\Pi,\iota_\alpha,\sigma) e$$

$$+\theta^T g(\Pi,\iota_\alpha,\delta) e \} dt$$

subject to

$$\eta^T \left[h_\lambda(\Pi,\varsigma) - Q^0 z_\lambda(\Pi,\gamma) \right] + \rho^T f_\lambda(\Pi,\iota_\alpha,\sigma) + \theta^T g_\lambda(\Pi,\iota_\alpha,\delta)$$

$$-D_\alpha \left[\rho^T f_{\lambda_\alpha}(\Pi,\iota_\alpha,\sigma) + \theta^T g_{\lambda_\alpha}(\Pi,\iota_\alpha,\delta) \right] = 0, \tag{11}$$

$$\eta^T \left[h_\pi(\Pi,\varsigma) - Q^0 z_\pi(\Pi,\gamma) \right] + \rho^T f_\pi(\Pi,\iota_\alpha,\sigma) + \theta^T g_\pi(\Pi,\iota_\alpha,\delta) = 0, \tag{12}$$

$$\iota(t_0) = \lambda_0, \quad \iota(t_1) = \lambda_1, \tag{13}$$

$$\eta \geq 0, \quad e^T \eta = 1, \quad e = (1,....1) \in \mathbb{R}^p, \tag{14}$$

$$\rho^T f(\Pi,\iota_\alpha,\sigma) \geqq 0, \tag{15}$$

$$g(\Pi,\iota_\alpha,\delta) = 0. \tag{16}$$

Definition 8. *A feasible point* $(\bar{\iota}, \bar{\kappa}, \bar{\eta}, \bar{\rho}, \bar{\theta}, \bar{\varsigma}, \bar{\gamma}, \bar{\sigma}, \bar{\delta}) \in \mathcal{D}_m$ *is named as a robust weak efficient solution to* $(m\mathcal{D} - \mathcal{P})$*, if there does not exist* $(\iota, \kappa, \eta, \rho, \theta, \varsigma, \gamma, \sigma, \delta) \in \mathcal{D}_m$ *fulfilling*

$$\int_S \{ \left[h(\Pi, \bar{\varsigma}) - Q^0 z(\Pi, \bar{\gamma}) \right] + \rho^T f(\Pi, \bar{\iota}_\alpha, \bar{\sigma}) e + \theta^T g(\Pi, \bar{\iota}_\alpha, \bar{\delta}) e \} dt$$

$$< \int_S \{ \left[h(\Pi, \bar{\varsigma}) - Q^0 z(\Pi, \bar{\gamma}) \right] + \rho^T f(\Pi, \iota_\alpha, \bar{\sigma}) e + \theta^T g(\Pi, \iota_\alpha, \bar{\delta}) e \} dt.$$

In the following, we establish a robust weak-type duality theorem for $(\mathcal{P})$.

Theorem 2 (Robust weak duality theorem). *Let* $(\bar{\lambda}, \bar{\pi})$ *and* $(\bar{\iota}, \bar{\kappa}, \bar{\eta}, \bar{\rho}, \bar{\theta}, \bar{\varsigma}, \bar{\gamma}, \bar{\sigma}, \bar{\delta})$ *be the robust feasible solutions of* $(\mathcal{P})$ *and* $(m\mathcal{D} - \mathcal{P})$*, respectively. Assume that* $\max_{\varsigma \in \mathcal{G}} h(\bar{\Upsilon}, \varsigma) = h(\bar{\Upsilon}, \bar{\varsigma})$ *and* $\min_{\gamma \in Q} z(\bar{\Upsilon}, \gamma) = z(\bar{\Upsilon}, \bar{\gamma})$*. Further, if* $\int_S \bar{\eta}^T \left[h(., \bar{\varsigma}) - Q^0 z(., \bar{\gamma}) \right] dt$*,* $\int_S \bar{\rho}^T f(., \bar{\sigma}) dt$ *and* $\int_S \bar{\theta}^T g(., \bar{\delta}) dt$ *are convex at* $(\bar{\iota}, \bar{\kappa})$*, then the following inequality cannot hold:*

$$\int_S \left[h(\bar{\Upsilon}, \bar{\varsigma}) - Q^0 z(\bar{\Upsilon}, \bar{\gamma}) \right] dt$$

$$< \int_S \{ \left[h(\Pi, \bar{\varsigma}) - Q^0 z(\Pi, \bar{\gamma}) \right] + \bar{\rho}^T f(\Pi, \bar{\iota}_\alpha, \bar{\sigma}) e + \bar{\theta}^T g(\Pi, \bar{\iota}_\alpha, \bar{\delta}) e \} dt.$$

Theorem 3 (Robust strong duality theorem). *Let* $(\bar{\lambda}, \bar{\pi})$ *be a robust weak efficient solution to* $(\mathcal{P})$. *Consider that* $\max_{\varsigma \in \mathcal{G}}\{h(\bar{\Upsilon}, \varsigma) - Q^0 \min_{\gamma \in Q} z(\bar{\Upsilon}, \gamma)\} = h(\bar{\Upsilon}, \bar{\varsigma}) - Q^0 z(\bar{\Upsilon}, \bar{\gamma})$ *and the constraint qualification conditions hold for* $(\mathcal{P})$. *Then,* $\bar{\eta} \in \mathbb{R}^p$, $\bar{\rho} = (\bar{\rho}_l(t)) \in \mathbb{R}^m_+$, $\bar{\theta} = (\bar{\theta}_b(t)) \in \mathbb{R}^n$ *exist as the piecewise smooth functions, and* $\bar{\sigma} \in \mathcal{T}$, $\bar{\delta} \in \mathcal{M}$, $\bar{\varsigma} \in \mathcal{G}$ $\bar{\gamma} \in Q$ *as the parameters of uncertainty such that* $(\bar{\lambda}, \bar{\pi}, \bar{\eta}, \bar{\rho}, \bar{\theta}, \bar{\varsigma}, \bar{\gamma}, \bar{\sigma}, \bar{\delta})$ *is a robust feasible solution to* $(m\mathcal{D} - \mathcal{P})$. *Moreover, if Theorem 3.1 holds, then* $(\bar{\lambda}, \bar{\pi}, \bar{\eta}, \bar{\rho}, \bar{\theta}, \bar{\varsigma}, \bar{\gamma}, \bar{\sigma}, \bar{\delta})$ *is a robust weak efficient solution to* $(m\mathcal{D} - \mathcal{P})$.

Theorem 4 (Robust strict converse duality theorem). *Let* $(\bar{\iota}, \bar{\kappa}, \bar{\eta}, \bar{\rho}, \bar{\theta}, \bar{\varsigma}, \bar{\gamma}, \bar{\sigma}, \bar{\delta})$ *be a robust feasible solution in* $(m\mathcal{D} - \mathcal{P})$. *Consider that* $\max_{\varsigma \in \mathcal{G}}\{h(\bar{\Upsilon}, \varsigma) - Q^0 \min_{\gamma \in Q} z(\bar{\Upsilon}, \gamma)\} = h(\bar{\Upsilon}, \bar{\varsigma}) - Q^0 z(\bar{\Upsilon}, \bar{\gamma})$ *and* $\int_S \bar{\eta}^T \left[h(., \bar{\varsigma}) - Q^0 z(., \bar{\gamma})\right] dt$, $\int_S \bar{\rho}^T f(., \bar{\sigma}) dt$ *and* $\int_S \bar{\theta}^T g(., \bar{\delta}) dt$ *are strictly convex at* $(\bar{\iota}, \bar{\kappa})$. *If* $(\bar{\lambda}, \bar{\pi}) \in \mathcal{S}$ *such that*

$$\int_S \left[h(\bar{\Upsilon}, \bar{\varsigma}) - Q^0 z(\bar{\Upsilon}, \bar{\gamma})\right] dt$$

$$= \int_S \{ \left[h(\bar{\Pi}, \bar{\varsigma}) - Q^0 z(\bar{\Pi}, \bar{\gamma})\right] + \bar{\rho}^T f(\bar{\Pi}, \bar{\iota}_\alpha, \bar{\sigma}) e + \bar{\theta}^T g(\bar{\Pi}, \bar{\iota}_\alpha, \bar{\delta}) e \} dt,$$

then, $(\bar{\lambda}, \bar{\pi})$ *is a robust weak efficient solution in* $(\mathcal{P})$.

- M. Ciontescu, **S. Treanţă**, *On some connections between interval-valued variational control problems and the associated inequalities,* Results in Control and Optimization, Vol. 12, 100300, 2023; DOI: 10.1016/j.rico.2023.100300; eISSN: 2666-7207.

Scopul principal al acestei lucrări este de a investiga relațiile dintre o clasă de probleme de optimizare cu valori pe interval și inegalitățile asociate. Concret, utilizând soluțiile inegalităților corespunzătoare (slabe, defalcate), stabilim unele rezultate de existență a soluțiilor LU-optimale (puternice) pentru problema de optimizare cu valori pe interval considerată. În acest sens, proprietatea de LU-convexitate a funcționalelor de tip integrală joacă un rol important. De asemenea, este prezentată și o aplicație ilustrativă care verifică afirmațiile și rezultatele teoretice.

$$(P) \qquad \min_{(y(\cdot),z(\cdot))} \int_M g\left(t, y(t), y_\gamma(t), z(t)\right) d\zeta$$

subject to

$$K_\delta\left(t, y(t), y_\gamma(t), z(t)\right) \leq 0, \quad \delta = \overline{1,l}$$

$$T_\eta\left(t, y(t), y_\gamma(t), z(t)\right) := \frac{\partial y}{\partial t^\gamma}(t) - Q_\eta\left(t, y(t), z(t)\right) = 0, \quad \eta = \overline{1,r}$$

$$y|_{\partial M} = y_0 = \text{given},$$

where $t \in M, g : M \times Y \times Y \times Z \mapsto \mathcal{F}$, $K_\delta : M \times Y \times Y \times Z \mapsto \mathbb{R}$ and $T_\eta : M \times Y \times Y \times Z \mapsto \mathbb{R}$ are C^1-class functions.

Now, in order to obtain some characterization results for the interval-valued optimization problem (P), we state the following inequalities:

(i) find $(y^0, z^0) \in S$ such that there exists no $(y, z) \in S$, satisfying

$$(I) \qquad \int_M \left\{ g_y^L\left(\pi^0\right) + g_y^U\left(\pi^0\right) \right\} \left(y - y^0\right) d\zeta + \int_M \left\{ g_z^L\left(\pi^0\right) + g_z^U\left(\pi^0\right) \right\} \left(z - z^0\right) d\zeta + \int_M \left\{ g_{y_\gamma}^L\left(\pi^0\right) + g_{y_\gamma}^U\left(\pi^0\right) \right\} D_\gamma \left(y - y^0\right) d\zeta \leq 0;$$

(ii) find $(y^0, z^0) \in S$ such that there exists no $(y, z) \in S$, satisfying

$$(WI) \qquad \int_M \left\{ g_y^L\left(\pi^0\right) + g_y^U\left(\pi^0\right) \right\} \left(y - y^0\right) d\zeta + \int_M \left\{ g_z^L\left(\pi^0\right) + g_z^U\left(\pi^0\right) \right\} \left(z - z^0\right) d\zeta + \int_M \left\{ g_{y_\gamma}^L\left(\pi^0\right) + g_{y_\gamma}^U\left(\pi^0\right) \right\} D_\gamma \left(y - y^0\right) d\zeta < 0;$$

(iii) find $(y^0, z^0) \in S$ such that, for all $(y, z) \in S$, the inequalities

$$(SI) \qquad \int_M g_y^L\left(\pi^0\right)\left(y - y^0\right) d\zeta + \int_M g_z^L\left(\pi^0\right)\left(z - z^0\right) d\zeta + \int_M g_{y_\gamma}^L\left(\pi^0\right) D_\gamma\left(y - y^0\right) d\zeta > 0,$$

$$\int_M g_y^U\left(\pi^0\right)\left(y - y^0\right) d\zeta + \int_M g_z^U\left(\pi^0\right)\left(z - z^0\right) d\zeta + \int_M g_{y_\gamma}^U\left(\pi^0\right) D_\gamma\left(y - y^0\right) d\zeta > 0$$

are satisfied.

Theorem 3.1. *Let* $(y^0, z^0) \in S$ *be a solution to* (I) *and functional* $\int_M g(\pi)d\zeta$ *is LU-convex at* $(y^0, z^0) \in S$. *Then* $(y^0, z^0) \in S$ *is an LU-optimal solution to* (P).

Theorem 3.2. *Let* $(y^0, z^0) \in S$ *be an LU-optimal solution of* (P) *and* $\int_M -g(\pi)d\zeta$ *is strictly LU-convex at* $(y^0, z^0) \in S$. *Then* $(y^0, z^0) \in S$ *is a solution of* (I).

Theorem 3.3. *Let* $(y^0, z^0) \in S$ *fulfils* (SI) *and* $\int_M g(\pi)d\zeta$ *is LU-convex at* $(y^0, z^0) \in S$. *Then* $(y^0, z^0) \in S$ *is a strong LU-optimal solution for* (P).

Theorem 3.4. *Let* $(y^0, z^0) \in S$ *be a strong LU-optimal solution of* (P) *and* $\int_M -g(\pi)d\zeta$ *is LU-convex at* $(y^0, z^0) \in S$. *Then* $(y^0, z^0) \in S$ *satisfies* (SI).

Theorem 3.5. *Consider* $(y^0, z^0) \in S$ *is an LU-optimal solution of* (P) *and* $\int_M -g(\pi)d\zeta$ *is LU-convex at* $(y^0, z^0) \in S$. *Then* $(y^0, z^0) \in S$ *solves the weak variational control inequality* (WI).

Theorem 3.6. *Let* $(y^0, z^0) \in S$ *solves the weak variational control inequality* (WI) *and* $\int_M g(\pi)d\zeta$ *is strictly LU-convex at* $(y^0, z^0) \in S$. *Then* $(y^0, z^0) \in S$ *is an LU-optimal solution of* (P).

Illustrative application. Let us minimize the interval-valued cost functional (which represents the mass of M with interval-valued density

$$g\left(t, y(t), y_\gamma(t), z(t)\right) = \left[g^L\left(t, y(t), y_\gamma(t), z(t)\right), g^U\left(t, y(t), y_\gamma(t), z(t)\right)\right]$$
$$= \left[(z(t)-4)^2,\ z^2(t)\right],$$

that depends on the current point) given by

$$(VP1) \qquad \min_{(y(\cdot),z(\cdot))} \int_M g(\pi)dt^1dt^2$$
$$= \left[\int_M g^L(\pi)dt^1dt^2, \int_M g^U(\pi)dt^1dt^2\right]$$
$$= \left[\int_M (z-4)^2dt^1dt^2, \int_M z^2dt^1dt^2\right]$$

subject to

$$\frac{\partial y}{\partial t^1} = \frac{\partial y}{\partial t^2} = 3 - z,$$
$$81 - y^2 \leq 0,$$
$$y(0,0) = 6, \quad y(3,3) = 8.$$

$M = [0,3]^2 = [0,3] \times [0,3] \subset \mathbb{R}^2$

4. Concluzii

- Potrivit planului de lucru, ce descrie sarcinile specifice științifice și neștiințifice legate de obiectivele proiectului, suntem în conformitate;
- Lucrările trimise spre publicare (sau în lucru) continuă să trateze subiecte menționate ca și obiective specifice ale proiectului de cercetare.

Vă mulțumesc pentru atenție!