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CS Dr. Virgil V. BĂRAN

Calcul simbolic pentru rezolvarea problemelor de mai multe corpuri în mecanica cuantică

– Aproximări şi alte aplicaţii ale matematicii în fizică şi inginerie –

– Raport intermediar –

Director de proiect
CS I Dr. Doru Sabin DELION

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Introducere

Modelul de clusterizare α pentru nucleele atomice a fost propus pentru a explica stabilitatea nucleelor usore $4n$ [1, 2]. Principala dificultate teoretica este legata de puternicele efecte de antisimetrizare intre nucleonii ce formeaza structurile de tip α. Acestea au fost incluse in diverse modele microscopice de clustering α [3, 4, 5, 6, 7, 8, 9, 10, 11].

In acelasi timp, o versiune simplificata in care se trateaza perechile de protoni si neutroni in aproximatia bozonica a avut succes in a explica efectul de staggering par-impar al energiilor de legatura [12]. In nuclee medii si grele clustering-ul α poate fi legat experimental de fenomenul de dezintegrare α [13]. O componenta de clustering α a campului mediu este necesara, pe langa baza uniparticula standard, pentru a descrie valorile absolute ale largimilor de dezintegrare α[14, 15]. Acest fapt este legat de aparitia particulelor α numai la densitati nucleare relativ scazute [16], situatie ce se realizeaza pe suprafata nucleelor emitatoare α [17].

O situatie similara se regaseste in cazul configuratiilor speciale precum starea Hoyle in ^{12}C, care poate fi vazuta ca o grupare slab legata de trei particule α condensate, ca bozoni, in starea $0S$ a campului lor mediu. Intelegerea dinamicii clusterilor α in astfel de situatii a fost semnificativ imbunatatita de abordarea recenta THSR [18]. O condensare de tip bozonic insa nu este realizata pentru sisteme finite de fermioni clusterizati, intrucat in general exista manifestari reziduale semnificative ale principiului de excluziune Pauli. Cu toate acestea, o proprietate utila a abordarii THSR este aceea de a cuprinde cele doua limite opuse, aceea de determinant Slated pur, precum si aceea in care cele doua particule α sunt suficient de departate pentru a putea permite neglijarea efectelor principiului Pauli [19].

De asemenea, recent a fost propus modelul Quartet Condensation Model (QCM) pentru studiul corelatiilor de quarteting in nuclee $N = Z$ [24, 25], acesta fiind dezvoltat ulterior in Refs. [26, 27, 28, 29, 30, 31] la cazul pairingului izoscalar si pentru nuclee $N > Z$. Modele microscopice de quartet mai generale au fost de asemenea recent dezvoltate [32, 33, 34, 35, 36, 37]. In abordarile de tip quarteting, caramizile de baza ale modelelor nu sunt perechile Cooper standard, ci structuri de patru corpuri compuse din doi protoni si doi neutroni cuplati la izospin total $T = 0$ si la moment cinetic $J = 0$, denumite "quarteti de tip α". Abordarea QCM s-a dovedit a fi foarte precisa pentru descrierea corelatiilor prezente in starea fundamentala a nucleelor $N = Z$. Efectele de antisimetrizare sunt semnificative in aceste configuratii, astfel incat realizarea unui condensat α, in sensul mentionat anterior, este o problema deschisa.

Astfel, in cele ce urmeaza, vom folosi termenii de "condensat de perechi" si "condensat de quarteti" pentru a indica starile de proba de tip BCS proiectat (PBCS) si de tip QCM din ecuatiile (2.2) si (2.10).

Este de notat ca exista dificultati intrinseci si in descrierea corelatiilor relativ simple de pairing, ceea ce a condus la eforturi semnificative dedicate formularii de descrieri apriximative, precum RPA [39] si metode de tip coupled clusters [40, 41, 42]. Recent, un tratament aproximativ interesant pentru corelatiile de pairing a fost dezvoltat in [43], pornind de la reformularea condensatului PBCS in termeni de excitatii particula-gol. In cele ce urmeaza vom generaliza aceste idei la cazul mai complicat al corelatiilor de quartet. Vom arata ca descrierea in termeni de excitatii particula-gol este naturala pentru ambele modele, PBCS si QCM. Suntem astfel motivati sa gasim reprezentarea condensatului de quarteti QCM in termeni de excitatii particula-gol fata de starea Hartree-Fock (a se vedea ecuatiile (2.14-2.15) mai jos).

De asemenea, vom introduce o noua abordare hibrida fermionic-bozonica, ce poate fi aplicata atat corelatiilor de pairing cat si celor de quarteting pentru simplificarea studiului corelatiilor in starea fundamentala. Primul pas al metodei presupune reformularea condensatului fata de starea corelata Hartree-Fock, spre deosebire de starea de vid $|0\rangle$. Acest fapt asigura ca o cantitate semnificativa de corelatii sunt deja incorporate, daca trecem la grade de libertate bozonice dar pastram aceeasi structura a starii de proba. In aceste conditii, cea mai simpla corespondenta de la operatori de perechi fermionice la bozoni ofera o buna descriere a proprietatilor starii fundamentale atat in cazul de pairing cat si de quarteting, fata de cazul exact fermionic.

Cu toate ca ideea de baza de a considera corelatiile de quarteting intr-un formalism bozonic (a se vedea [12], [45, 46, 47, 48]), precum si aceea de a considera aproximatia bozonica pentru excitatiile particula-gol nu sunt noi [49], abordarea in doua etape nu a mai fost implementata in felul prezentat anterior.

Este de remarcat ca in ambele cazuri, de pairing si quarteting, formalismul este asemanator din punct de vedere structural, ducand la aceeasi forma functionala a energiei condensatului bozonic, pana la factori de forma. In acest sens, o descriere unificata a corelatiilor de pairing si a celor de quarteting (semnificativ mai complicate) este posibila.

Cadrul teoretic

Consideram un model avand un numar N_{lev} de nivele dublu degenerate $i, \bar{i}$ de energii uniparticula ϵ_i, unde starea fundamentala a Hamiltonianului

$$H = \sum_{i=1}^{N_{\rm lev}} \epsilon_i \left(c_i^\dagger c_i + c_{\bar{i}}^\dagger c_{\bar{i}} \right) + \sum_{i,j=1}^{N_{\rm lev}} V_{ij} P_i^\dagger P_j \ , \tag{2.1}$$

este considerata a fi condensatul PBCS de n_p perechi,

$$|PBCS\rangle = \left(\Gamma^\dagger(x) \right)^{n_p} |0\rangle \ . \tag{2.2}$$

O pereche coerenta este o superpozitie de perechi uniparticula $P_i^\dagger = c_i^\dagger c_{\bar{i}}^\dagger$,

$$\Gamma^\dagger(x) = \sum_{i=1}^{N_{\rm lev}} x_i P_i^\dagger \ , \tag{2.3}$$

iar $|0\rangle$ este starea de vid fara particula.

Urmand [50, 43], in loc sa exprimam starea $|PBCS\rangle$ fata de vidul $|0\rangle$, putem gasi o expresie echivalenta in termeni de starea Hartree-Fock

$$|{\rm HF}\rangle = \left(\prod_{i=1}^{n_p} P_i^\dagger\right)|0\rangle \ . \tag{2.4}$$

Descompunem mai intai perechea coerenta in componente deasupra si dedesubtul nivelului Fermi

$$\Gamma^\dagger(x) = \sum_{i=1}^{n_p} x_i P_i^\dagger + \sum_{i=n_p+1}^{N_{\rm lev}} x_i P_i^\dagger \equiv \Gamma_h^\dagger(x) + \Gamma_p^\dagger(x) \tag{2.5}$$

Actiuna componentei pe subspatiul de goluri de argumente x pe vidul $|0\rangle$ poate fi exprimata in termeni de actiunea perechii coerente de argumente inverse $1/x$ pe starea Hartree-Fock. Astfel, poate fi demonstrata reformularea condensatului de perechi

$$|PBCS\rangle = n_p! \cdot \Pi_1 \sum_{j=0}^{n_p} \frac{1}{(j!)^2}\left(\Gamma_p^\dagger(x)\ \Gamma_h\left(\frac{1}{x}\right)\right)^j |{\rm HF}\rangle, \tag{2.6}$$

unde $\Pi_1 = x_1 x_2 \cdots x_{n_p}$.

Aceasta abordare poate fi generalizata la corelatii de quartet. Astfel, consideram Hamiltonianul de pairing izovector

$$H = \sum_{i=1}^{N_{\rm lev}} \epsilon_i \left(N_{i,\pi} + N_{i,\nu}\right) + \sum_{\tau=0,\pm1} \sum_{i,j=1}^{N_{\rm lev}} V_{ij} P_{i,\tau}^\dagger P_{j,\tau} \ , \tag{2.7}$$

unde $\tau = 0, \pm1$ este proiectia izospinului. In cadrul QCM, sunt definite perechi colective de tip $\pi\pi$, $\nu\nu$ si $\pi\nu$

$$\Gamma_\tau^\dagger(x) \equiv \sum_{i=1}^{N_{\rm lev}} x_i P_{\tau,i}^\dagger \ , \tag{2.8}$$

Un quartet colectiv este construit prin cuplajul a doua perechi colective la izospin total $T = 0$

$$Q^\dagger \equiv \left[\Gamma^\dagger \Gamma^\dagger\right]_{S=0}^{T=0} \equiv 2\Gamma_1^\dagger \Gamma_{-1}^\dagger - \left(\Gamma_0^\dagger\right)^2 \ . \tag{2.9}$$

Starea fundamentala a Hamiltonianului (2.7) este descrisa ca un condensat de astfel de quarteti

$$|\Psi_q(x)\rangle = \left(Q^\dagger\right)^q |0\rangle \ , \tag{2.10}$$

unde q este numarul de quarteti. Amplitudinile de amestec x_i definesc structura starii, ele fiind determinate numeric prin minimizarea valorii medii a Hamiltonianului cu constrangerea de norma unitate.

In analogie cu cazul de pairing, in loc sa exprimam starea de condensat de quarteti in termeni de vidul $|0\rangle$ vacuum, putem sa gasim o forma echivalenta in termeni de starea Hartree-Fock, in acest caz data de

$$|\mathrm{HF}\rangle = \left(\prod_{i=1}^{q} P^{\dagger}_{1,i} P^{\dagger}_{-1,i} \right) |0\rangle \ . \tag{2.11}$$

Perechile colective pot fi descompuse in componente dedesubtul si deasupra nivelului Fermi

$$\Gamma^{\dagger}_{\tau}(x) = \sum_{i=1}^{q} x_i P^{\dagger}_{\tau,i} + \sum_{i=n_p+1}^{N_{\mathrm{lev}}} x_i P^{\dagger}_{\tau,i} \equiv \Gamma^{\dagger}_{\tau,h}(x) + \Gamma^{\dagger}_{\tau,p}(x) \ . \tag{2.12}$$

In consecinta, quartetul colectiv se descompune ca

$$\begin{aligned} Q^{\dagger}(x) &= 2\Gamma^{\dagger}_{1}\Gamma^{\dagger}_{-1} - \left(\Gamma^{\dagger}_{0}\right)^2 \\ &= 2\Gamma^{\dagger}_{1,h}\Gamma^{\dagger}_{-1,h} - \left(\Gamma^{\dagger}_{0,h}\right)^2 + 2\Gamma^{\dagger}_{1,p}\Gamma^{\dagger}_{-1,p} - \left(\Gamma^{\dagger}_{0,p}\right)^2 \\ &+ 2\left(\Gamma^{\dagger}_{1,p}\Gamma^{\dagger}_{-1,h} + \Gamma^{\dagger}_{-1,p}\Gamma^{\dagger}_{1,h} - \Gamma^{\dagger}_{0,p}\Gamma^{\dagger}_{0,h}\right) \\ &\equiv Q^{\dagger}_{h}(x) + Q^{\dagger}_{p}(x) + 2\left[\Gamma^{\dagger}_{p}(x)\Gamma^{\dagger}_{h}(x)\right] \ . \end{aligned} \tag{2.13}$$

Un calcul similar cu cel din cazul corelatiilor de pairing, ce consta in evaluarea efectelor perechilor colective de argumente inverse $1/x$, duce la expresia condensatului de quarteti sub forma de excitatii particula-gol fata de starea Hartree-Fock

$$\begin{aligned} |\Psi_q\rangle = 2^q \ q! \ \Pi_2 \sum_{a=0}^{q}\sum_{b=0}^{q} \lambda_{ab} \left(Q^{\dagger}_{p}(x) Q_h\left(\frac{1}{x}\right)\right)^a \\ \times \left[\Gamma^{\dagger}_{p}(x)\Gamma_h\left(\frac{1}{x}\right)\right]^b |\mathrm{HF}\rangle \ , \end{aligned} \tag{2.14}$$

unde

$$\begin{aligned} \lambda_{ab} = \frac{1}{2^a \ b!} \sum_{r=\mathrm{Max}(0,N_{ab}-q)}^{a} \frac{(q-N_{ab})_{a-r}}{2^r (a-r)!(r!)^2} \\ \times \frac{\Gamma\left(\frac{3}{2}+q-r\right)}{\Gamma\left(\frac{3}{2}+N_{ab}-r\right)} \ , \end{aligned} \tag{2.15}$$

si $\Pi_2 = x_1^2 x_2^2 \cdots x_q^2$. In formula precedenta $N_{ab} = 2a + b$ este numarul de perechi din fiecare termen, $\Gamma(z)$ este functia Gamma iar $(z)_k = z(z-1)...(z-k)$ este simbolul Pochammer. De asemenea, folosim notatia

$$\left[\Gamma^{\dagger}_{p}(x)\Gamma_h\left(\frac{1}{x}\right)\right] \equiv \sum_{\tau=\pm1,0} \Gamma^{\dagger}_{\tau,p}(x)\Gamma_{\tau,h}\left(\frac{1}{x}\right) \ . \tag{2.16}$$

Cu notatia $|\Psi_q\rangle = \Pi_2 \mathcal{O}_q |\mathrm{HF}\rangle$, cateva expresii particulare pentru $\mathcal{O}_q$ sunt

$$
\begin{aligned}
\mathcal{O}_1 &= 2\left[\Gamma_p^\dagger\Gamma_h\right] + \frac{1}{3}\left(Q_p^\dagger Q_h\right) + 3 \\
\mathcal{O}_2 &= 4\left[\Gamma_p^\dagger\Gamma_h\right]^2 + 20\left[\Gamma_p^\dagger\Gamma_h\right] + \frac{1}{30}\left(Q_p^\dagger Q_h\right)^2 + \frac{4}{5}\left[\Gamma_p^\dagger\Gamma_h\right]\left(Q_p^\dagger Q_h\right) + 2\left(Q_p^\dagger Q_h\right) + 30 \\
\mathcal{O}_3 &= 8\left[\Gamma_p^\dagger\Gamma_h\right]^3 + 84\left[\Gamma_p^\dagger\Gamma_h\right]^2 + 420\left[\Gamma_p^\dagger\Gamma_h\right] + \frac{1}{630}\left(Q_p^\dagger Q_h\right)^3 + \frac{3}{35}\left[\Gamma_p^\dagger\Gamma_h\right]\left(Q_p^\dagger Q_h\right)^2 \\
&\quad - \frac{99}{70}\left(Q_p^\dagger Q_h\right)^2 + \frac{12}{7}\left[\Gamma_p^\dagger\Gamma_h\right]^2\left(Q_p^\dagger Q_h\right) + 12\left[\Gamma_p^\dagger\Gamma_h\right]\left(Q_p^\dagger Q_h\right) + 114\left(Q_p^\dagger Q_h\right) + 630\ .
\end{aligned}
\tag{2.17}
$$

Expresiile pentru $\mathcal{O}_1$ si $\mathcal{O}_2$ au fost de asemenea construite termen cu termen cu ajutorul pachetului software simbolic Cadabra 2 [51, 52, 53].

Expresiile din ecuatiile (2.6) si (2.14)-(2.15) sunt punctul de plecare pentru tratamentul bozonic al corelatiilor de pairing si quarteting. Ca prim pas, pentru exprimarea Hamiltonianului in termeni de grade de libertate de particula-gol, introducem indici de particula $(i, j, k, \dots)$ si de gol $(a, b, c, \dots)$. Pentru subspatiul de goluri, notam operatorul de creare de perechi $\tilde{P}_a^\dagger \equiv P_a$. Descompunerea Hamiltonianului de pairing al ecuatiei (2.1) in componente de particule si goluri este

$$
\begin{aligned}
H &= \sum_{a=1}^{n_p}(2\epsilon_a + V_{aa}) + \sum_{a=1}^{n_p}(-\epsilon_a - V_{aa})\tilde{N}_a + \sum_{i=n_p+1}^{N_{\text{lev}}} \epsilon_i N_i \\
&+ \sum_{a,b=1}^{n_p} V_{ab}\tilde{P}_a^\dagger \tilde{P}_b + \sum_{i,j=n_p+1}^{N_{\text{lev}}} V_{ij} P_i^\dagger P_j + \sum_{a=1}^{n_p}\sum_{j=n_p+1}^{N_{\text{lev}}} V_{ai}\left(\tilde{P}_a P_i + P_i^\dagger \tilde{P}_a^\dagger\right)\ .
\end{aligned}
\tag{2.18}
$$

unde operatorul numar de goluri este $\tilde{N}_a = 2 - N_a$. Pentru cazul de pairing izovector, introducem in mod similar operatorii de perechi de goluri pentru fiecare proiectie de izospin, $\tilde{P}_{\tau,a}^\dagger \equiv P_{\tau,a}$, precum si operatorul corespunzator numarului total de goluri $\tilde{N}_{0,a} = 4 - N_{0,a}$. Astfel, Hamiltonianul ecuatiei (2.7) se descompune ca

$$
\begin{aligned}
H &= \sum_{a=1}^{q}(4\epsilon_a + 3V_{aa}) + \sum_{a=1}^{q}(-\epsilon_a - \frac{3}{2}V_{aa})\tilde{N}_{0,a} + \sum_{i=q+1}^{N_{\text{lev}}} \epsilon_i N_{0,i} \\
&+ \sum_{a,b=1}^{q} V_{ab} \sum_{\tau=\pm1,0} \tilde{P}_{\tau,a}^\dagger \tilde{P}_{\tau,b} + \sum_{i,j=q+1}^{N_{\text{lev}}} V_{ij} \sum_{\tau=\pm1,0} P_{\tau,i}^\dagger P_{\tau,j} \\
&+ \sum_{a=1}^{q}\sum_{j=q+1}^{N_{\text{lev}}} V_{ai} \sum_{\tau=\pm1,0}\left(\tilde{P}_{\tau,a} P_{\tau,i} + P_{\tau,i}^\dagger \tilde{P}_{\tau,a}^\dagger\right)\ .
\end{aligned}
\tag{2.19}
$$

In cadrul aproximatiei bozonice, definim corespondenta de la operatorii de pereche la operatori bozonici

$$
P_i^\dagger \to p_i^\dagger\ ,\ \tilde{P}_a^\dagger \to h_a^\dagger\ ,\ |\text{HF}\rangle \to |0)\ . \tag{2.20}
$$

unde $p_i|0) = 0$ and $h_a|0) = 0$, impreuna cu corespondenta $\tilde{N}_a \to \mathcal{N}_a$, $N_i \to \mathcal{N}_i$, unde

$\mathcal{N}_i|0) = 0$ si $\mathcal{N}_a|0) = 0$. Acesti operatori formeaza o algebra bozonica,

$$\begin{aligned} \left[p_i, p_j^\dagger\right] &= \delta_{ij}\pi_j, \left[h_a, h_b^\dagger\right] = \delta_{ab}\eta_b, \left[p_i, h_j^\dagger\right] = 0 \ , \\ \left[\mathcal{N}_i, p_j^\dagger\right] &= 2\delta_{ij}p_j^\dagger, \left[\mathcal{N}_a, h_b^\dagger\right] = 2\delta_{ab}h_b^\dagger \end{aligned} \tag{2.21}$$

unde coeficientii π_i si η_j sunt scalari. In continuare, definim operatorii bozonici colectivi

$$\mathcal{H}^\dagger(y) \equiv \sum_{a=1}^{n_p} y_a h_a^\dagger \ , \ \mathcal{P}^\dagger(x) \equiv \sum_{i=n_p+1}^{N_{\rm lev}} x_i p_i^\dagger \ . \tag{2.22}$$

cu ajutorul carora construim o stare de proba bozonica structural identica cu cea a condensatului PBCS fermionic

$$|\psi(x,y)\rangle \equiv \sqrt{\chi}\sum_{n=0}^{n_p} \frac{1}{(n!)^2}\left(\mathcal{P}^\dagger(x)\ \mathcal{H}^\dagger(y)\right)^n |0) \ , \tag{2.23}$$

unde χ este o constanta de normare. Media Hamiltonianului pe starea de proba, in cazul bozonic, poate fi gasita analitic ca

$$\begin{aligned} \langle H_b\rangle &= (\mathcal{H}_{hh}S_p + \mathcal{H}_{pp}S_h)\cdot f_1(S_{ph}) + \mathcal{H}_{ph}\cdot f_2(S_{ph}) + E_0\cdot\nu(S_{ph}) \ , \\ \mathcal{H}_{hh} &= \sum_{a=1}^{n_p} 2\tilde{\epsilon}_a\eta_a y_a^2 + \sum_{a,b=1}^{n_p} V_{ab}y_a\eta_a y_b\eta_b \ , \\ \mathcal{H}_{pp} &= \sum_{i=n_p+1}^{N_{\rm lev}} 2\epsilon_i\pi_i x_i^2 + \sum_{i,j=n_p+1}^{N_{\rm lev}} V_{ij}x_i\pi_i x_j\pi_j \ , \\ \mathcal{H}_{ph} &= 2\sum_{a=1}^{n_p}\sum_{j=n_p+1}^{N_{\rm lev}} V_{ai}x_i\pi_i y_a\eta_a \ , \end{aligned} \tag{2.24}$$

in termeni de factorii de forma

$$f_1(z) = \sum_{n=1}^{n_p}\frac{nz^{n-1}}{(n!)^2} \ , \ f_2(z) = \sum_{n=0}^{n_p-1}\frac{z^n}{(n!)^2} \ , \nu(z) = \sum_{n=0}^{n_p}\frac{(z)^n}{(n!)^2} \tag{2.25}$$

si energia $E_0 = \sum_{a=1}^{n_p}(2\epsilon_a + V_{aa})$, unde $S_p = \sum_{i=n_p+1}^{N_{\rm lev}} x_i^2\pi_i$, $S_h = \sum_{a=1}^{n_p} y_a^2\eta_a, S_{ph} = S_pS_h$.

Energia starii fundamentale corespunde minimului functiei

$$E(x,y) \equiv \frac{\langle\psi(x,y)|H_b|\psi(x,y)\rangle}{\langle\psi(x,y)|\psi(x,y)\rangle} \ , \tag{2.26}$$

si poate fi calculata numeric prin minimizarea fata de amplitudinile de particula si gol x_i si y_a. Consideram doua alegeri pentru coeficientii comutatorilor din ecuatiile (2.21), si anume

1. cazul bozonic pur: $\eta_a = 1$, $\pi_i = 1$.

2. cazul bozonic renormalizat:

$$\begin{aligned} \eta_a &= 1 - \frac{1}{2}\langle\mathcal{N}_a\rangle = 1 - y_a^2\eta_a S_p f_1(S_{ph})/\nu(S_{ph}) \\ \pi_i &= 1 - \frac{1}{2}\langle\mathcal{N}_i\rangle = 1 - x_i^2\pi_i S_h f_1(S_{ph})/\nu(S_{ph}) \end{aligned} \tag{2.27}$$

Este de preferat o procedura de renormalizare pentru a lua in considerare in mod efectiv ocuparea maxima finita a unui nivel in acord cu principiul Pauli.

Aceeasi idee de baza este aplicabila si in cazul de pairing izovector. Introducem corespondenta de la fiecare operator de pereche de proiectie a izospinului data la un operator bozonic

$$P^{\dagger}_{\tau,i} \to p^{\dagger}_{\tau,i} \ , \ \tilde{P}^{\dagger}_{\tau,a} \to h^{\dagger}_{\tau,a} \ . \tag{2.28}$$

Consideram aproximatia de independenta a perechilor bozonice de proiectii diferite ale izospinului:

$$\left[p_{\tau,i}, p^{\dagger}_{\sigma,j}\right] = \delta_{\tau\sigma}\delta_{ij}\pi_j \ , \ \left[h_{\tau,a}, h^{\dagger}_{\sigma,b}\right] = \delta_{\tau\sigma}\delta_{ab}\eta_b \tag{2.29}$$

Aceasta alegere permite un tratament simplu al corelatiilor de pairing izovector, insa este de mentionat ca sunt neglijate efectele de overlap ale perechii proton-neutron cu perechile de aceeasi specie, luate in considerare in formalismul fermionic original prin comutatorii $\left[P_{0,i}, P^{\dagger}_{\pm1,j}\right] = \pm\delta_{ij}T_{\pm1,i}$, unde operatorii de izospin sunt $T_{\tau,i} = \left[c^{\dagger}_i c_i\right]^{J=0,T=1}_{M=0,\tau}$.

In cadrul acestei aproximatii, media Hamiltonianului izovector pe versiunea bozonica a starii de proba de quartet poate fi exprimata intr-o forma similara cu cea din cazul de pairing standard. Ca si in cazul PBCS, consideram ambele alegeri de relatii de comutare bozonice, cele pure si cele renormalizate.

Pe tematica prezentata mai sus a fost publicat articolul “Unified description of pairing and quarteting correlations within the particle-hole-boson approach”, Physical Review C 99, 064311, 7 Iunie 2019, autori V.V. Baran si D. S. Delion. De asemenea, o a doua lucrare este in curs de redactare. Rezultatele urmeaza a fi prezentate in cadrul conferintelor stiintifice nationale si internationale.

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